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Abstract

This study compares the predictive accuracy of the Shapley value, nucleolus and egalitarian solution concepts for a 5-player cooperative game. The cooperative game in the present study has been parameterized in order to provide a model for conservation negotiations between private landowners on the landscape scale. The paper develops a player's profile for the cooperative game that represents a typical landscape conservation problem. Algorithmic procedures are used to generate hypothetical allocations for the three tested solution concepts, which are then compared with the results from a series of laboratory experiments. The results show that participants were able to reach a core allocation in 44% of rounds. The egalitarian solution strictly dominates the payoffs predicted by the Shapley value and nucleolus for rounds in which participants are able to reach a core allocation. When a core allocation is reached, participants tend to allocate the aggregate payoff from the grand coalition in an equal manner.

Chapter 1

Introduction

Cooperative game theory can be utilized to model scenarios where two or more individuals have the potential to make decisions with a common interest. In a cooperative game, players can either act individually or form coalitions with other players. Successfully forming a coalition provides an aggregate payoff to the members of a coalition, which can be then allocated in order to produce a positive benefit to each player (Van Neumann and Morganstern, 1944). The set of payoff allocations that is strictly preferred to all other possible allocations forms the core of the cooperative game. However, the core may not necessarily contain a unique allocation, necessitating the use of solution concepts to compact the core to a single solution. This thesis compares the predictive accuracy of the Shapley value, nucleolus and egalitarian solution concepts by the use of an experimental economics approach.

The comparison of solution concepts in cooperative game theory using laboratory data has received extensive treatment in the current literature, even though research has been primarily focused on 3-player (Rapoport and Kahan, 1976; Michener et al, 1982, 1983; Bolton, Chatterjee and McGinn, 2003; Beimborn, 2013) and 4-player cooperative games (Michener et al, 1979, 1980, 1983). The absence of laboratory analyses for games of 5 or more players can be attributed to an increase in solution complexity as the number of players increases (Michener, Yuen and Geishenker, 1980). Therefore, a gap in the current literature exists regarding the predictive accuracy of solution concepts for games with a larger number of players.

Cooperative game theory models have been extensively applied to natural resource management problems. Examples include the distribution of water allocation licenses (Young et al, 1982; Tisdell and Harrison, 1992; Rosen and Sexton, 1993; Lejano and Davos, 1995, 1999; Loaciga, 2004), and the placement of terrestrial noxious facilities (Marchetti and Serra, 2003). However, there are

no current studies which model landscape-scale conservation efforts using cooperative game theory. It is accepted that conservation efforts across a landscape require cooperation between individual private landowners. Difficulties arise in providing a lucid incentive system to encourage landowners to bear the opportunity costs of participating in a wildlife corridor, land connecting two or more habitat zones. Cooperative game theory can be utilized to model conservation negotiations, with individual private landowners taking the role of players in the game. In the model, each landowner can choose to operate their land individually or form a wildlife corridor with one or more other landowners. Incentives for corridor formation are introduced by providing a positive net benefit to landowners who form part of a successful corridor. Solution concepts in cooperative game theory can be utilized in order to predict the behavior of landowners in landscape-scale conservation negotiations. Further, for the model to be more useful in analyzing real-world conservation problems, the number of players should be increased beyond the 3- and 4-player games prevalent in previous literature.

This thesis expands on previous work (Michener et al, 1979, 1982; Beimborn, 2013) by comparing the predictive accuracy of the Shapley value, nucleolus and egalitarian solution concepts for a 5-player cooperative game. Algorithmic procedures were used to generate hypothetical allocations for the Shapley value, nucleolus and egalitarian solutions for the parameterized cooperative game. In order to test the accuracy of the solution concepts, data was collected from a series of laboratory experiments that provided a model for landscape-scale conservation negotiations. The results demonstrated that equal distribution of the aggregate payoff from coalition formation dominated the allocations predicted by either the Shapley value or nucleolus solution concepts when an allocation within the core of the game was reached. However, the difficulty in reaching a core allocation rose in comparison with previous studies (Michener et al, 1979, 1982) as the number of players was increased.

This thesis proceeds by providing a detailed description of the basic principles of cooperative game theory in Chapter 2, and the three solution concepts tested in

the present study. In addition, Chapter 2 provides a review of the current literature where cooperative game theory has been applied to natural resource management problems and laboratory experiments. Chapter 3 details the methodology used in this study to generate the hypothetical allocations of each solution concept, in addition to the design of the experiments performed. Chapter 4 describes the results of the laboratory experiments, while Chapter 5 concludes with a discussion of the results and policy recommendations for conservation negotiations on the landscape scale.

Chapter 2

Landscape Conservation as an *N*-Person Cooperative Game

This chapter describes the principles of cooperative game theory and its application to natural resource management. The chapter begins with an overview of the principles of cooperative game theory. This is followed by a review of current literature on the application of cooperative game theory to environmental management problems. The chapter then provides a detailed description of studies where laboratory experiments have been utilized to analyze solution concepts in cooperative game theory.

2.1 Overview of Cooperative Game Theory

Game theory is used to model decision-making scenarios between two or more players where the actions of one player can influence the payoffs of another. As a result, the interests of the involved players could be conflicting (Van Neumann and Morganstern, 1944; Winston, 2004). The game becomes cooperative when sub-groups, or coalitions, of players have a common interest to align their decision-making choices. Players in an n-person cooperative game can choose to: (i) operate individually; (ii) enter a *coalition* comprising the subset S of N or (iii) enter a *grand coalition* containing all n players. Solutions to such games are often expressed in characteristic function form, taking into account the payoffs gained by all potential coalitions as well as individual decision makers (Lejano and Davos, 1995; Curiel, 1997).

2.1.1 Characteristic Function

The *characteristic function* for an n-person cooperative game quantifies the aggregate benefit received by all members in a coalition working together. In a cooperative game the characteristic function form is represented by the ordered set $\{N, v(S)\}$, where $v(S)$ is a characteristic function for the *coalition* S. Thus, a *coalition* of players can be specifically defined as a subset S of N. The

characteristic function for a group containing no players must be equal to zero, so $v(\emptyset) = 0$. Further, if n is the number of players in the set N, then the payoffs to the players when acting individually is a row vector $[v(\{1\}), v(\{2\}), \dots, v(\{n\})]$. Thus, the number of potential groups in the cooperative game is $2^n - 1$, excluding the *null coalition* containing no players (Von Neumann and Morganstern, 1944; Tisdell and Harrison, 1992; Curiel, 1997).

2.1.2 Solutions and the Core

A potential *solution* for a cooperative game is represented by the *reward vector* **x** = $[x_1, x_2, \dots, x_n]$, where x_i indicates the reward received by player *i*. A reward vector, or allocation **x** is a *core* solution for the cooperative game if **x** satisfies the following three conditions (Curiel, 1997; Winston, 2004).

Individual Rationality – the payoff to player *i* from the reward vector must be at least as great as the payoff that player *i* would gain from acting individually.

$$x_i \geq v(\{i\}), \forall\ i \in N \tag{2.1}$$

Group Rationality - the allocation **x** is in the core only if the total of the payoffs received by the players in coalition *S* is at least as great as $v(S)$, the value of the appropriate sub-coalition.

$$\sum_{i \in S} x_i \ \geq v(S) \tag{2.2}$$

Grand Coalition - the total payoff from a reasonable reward vector must equal the payoff that would be gained from the formation of a *grand coalition* containing all players.

$$v(N) = \sum_{i=1}^{n} x_i \tag{2.3}$$

The *core* of the game *v* is defined as the set of all allocations **x** that are un-dominated by other potential allocations in *N* (Gillies, 1959). Moving from any allocation within the core to an allocation outside the core would be impossible without making at least one individual player worse off. That is, the core represents the set of Pareto-efficient payoff allocations (Van Neumann and Morganstern, 1944; Roth, 1976; Tisdell and Harrison, 1992; Winston, 2004).

2.2 Solution Concepts

In the present study, three solution concepts are utilized in order to compact the range of allocations generated by the core to a single unique solution.

2.2.1 Shapley Value

The Shapley value allocates a payoff to each player proportional to the expected benefit that the player brings to each of her coalitions at the moment of entry (Shapley, 1953). In this way, payoffs are distributed fairly on the sole basis of what each player brings to her coalition. Shapley (1953) proved that for every characteristic function, there exists a unique reward vector that satisfies the following axioms.

<u>Axiom 1:</u> *The number assigned to a player is linked to the payoff that the player receives from the Shapley reward vector. That is, if the individual payoffs of two players are exchanged, their rewards from the Shapley value will also be exchanged.*

<u>Axiom 2:</u> *The payoffs received by players in a coalition must be at least as great as the value of that coalition according to the characteristic function.*

<u>Axiom 3:</u> *When a player brings no positive benefit to a coalition, the Shapley value will award that player a payoff of zero. That is, when* $v(S\text{-}\{i\}) = v(S; i \in S)$*, the payoff to player i according to the Shapley vector will be equal to zero.*

Axiom 4: *Shapley reward vectors are additive. When* **x** *and* ***y*** *are the Shapley values for the games v and w respectively, the Shapley value for the game (v + w) is equal to (****x*** *+* ***y****).*

Therefore for any game with characteristic function *v*, there exists a unique Shapley vector **x** within the core that satisfies axioms 1 to 4 (Shapley, 1953; Curiel, 1997; Winston, 2004). The reward x_i received by the *i*th player in the game *v* is then determined as follows (Shapley, 1953).

$$x_i = \sum_{all\ S\ s.t\ i\ \in S} \frac{(|S|-1)!\,(n-|S|)!}{n!}[v(S) - v(S-\{i\})] \tag{2.4}$$

Where *n* is the number of players in the game, and $|S|$ is the number of players in coalition *S*. Therefore, the reward that player *i* receives should be directly proportional to the expected contribution that *i* makes to the value of the coalition in existence at the moment of *i*'s arrival (Shapley, 1953; Tisdell and Harrison, 1992; Winston, 2004). That is, the level of bargaining power held by that player determines the payoff received.

2.2.2 Nucleolus

The second solution concept examined is the *nucleolus*. The nucleolus minimizes the greatest dissatisfaction experienced by the set of players to any potential allocation (Schmeidler, 1969; Maschler, Peleg and Shapley, 1979; Tisdell and Harrison, 1992). Let the set of players in the cooperative game be *N*, and the number of players be *n*. The *excess e* of a coalition *S* with respect to an imputation **x** is defined as follows.

$$e = v(S) - \sum_{i \in S} x_i\,, where\ S \in N \tag{2.5}$$

The excess represents the difference between the aggregate payoff to the coalition S and the sum of the payoffs each player would receive if they acted individually. The excess is inversely proportional to the desirability of the

solution vector **x** to coalition S. Therefore, the excess e represents the dissatisfaction of the coalition S to the outcome **x**.

The excess is utilized to estimate the nucleolus via a linear programming approach. At each step taken in the programming algorithm, all coalitions whose excess is equal to the critical value of e are removed and for the remaining coalitions the largest excess is estimated. The iterative process continues until any further iteration would leave the core empty. The solution that remains is the nucleolus. Thus, the nucleolus is estimated using a minimax approach (Schmeidler, 1969; Lejano and Davos, 1995; Peleg and Sudholter, 2007). The linear programming problem is outlined below.

$$\min e = v(S) - \sum_{i \in S} x_i \, , where\ S \in N \tag{2.6}$$

Subject to:

$$x_i + e_{\ crit} \geq v(\{i\}), \forall\ i \in N \tag{2.7}$$

$$\sum_{i \in S} x_i \ + e_{crit} \ \geq v(S) \tag{2.8}$$

$$\sum_{i=1}^{n} x_i = v(N) \tag{2.9}$$

The nucleolus has several desirable solution concept properties. Schmeidler (1969) demonstrated that, for any non-empty core, the nucleolus uniquely exists and is a member of the core. In addition, the nucleolus provides some indication of how the aggregate benefits should be fairly distributed (as the dissatisfaction levels of all coalitions are gradually minimized) between the members of a coalition.

2.2.3 Egalitarian Allocation

The third solution concept utilized in the present study is the *egalitarian allocation*. The egalitarian allocation assumes that the players in an *n*-player cooperative game will equally divide the grand coalition payoff *v(N)* to achieve minimum variance (Michener, Ginsberg and Yuen, 1979).

$$x_i = \frac{v(N)}{n}, \forall\ i\ \in N \tag{2.10}$$

While the egalitarian allocation may be mathematically simple in comparison to the Shapley value and nucleolus, the apparent equality and ease of calculation might make it easier for players to realize and adopt. This study will explore how each of these solution concepts compares in terms of optimization and experimental outcomes. To place the study in context, the next section examines how n-person cooperative game theory, and the associated solution concepts, have been applied to natural resource management.

2.3 Applications to Natural Resource Management

Cooperative game theory models are highly applicable to resource management problems, as they typically involve a number of agents with both conflicting and parallel interests. Further, positive outcomes in managing natural resources commonly rely on cooperative efforts between such agents. This section will provide a detailed overview of the currently available literature on cooperative game theory, specifically focusing on studies using optimization and experimental economics techniques.

2.3.1 Optimization Studies

Many studies have utilized optimization techniques to solve problems in cooperative game theory, with a focus on water allocation problems. Young, Okada and Hashimoto (1982) applied a cooperative game theoretic model to the

problem of equitable joint cost allocation. The super-additive requirements of the core prevented situations where total costs were increased but some agents were required to pay less than before.

In order to produce this efficient allocation, a joint cost function was used to represent a characteristic function for the game (Young, Okada and Hashimoto, 1982). The cost of participating in a coalition was assumed to be dependent on the number of members in the coalition. In addition, economies of scale cost savings provided an incentive for players to form coalitions. The optimal scenario in terms of cost efficiency was demonstrated to be a grand coalition consisting of all players in the game.

Young, Okada and Hashimoto (1982) suggested that difficulties arise in equitably allocating the joint costs between members of a coalition. The purpose of equitable allocation was to provide an additional incentive for the formation of a grand coalition. In other words, if a player feels that their cost share is not equitable compared to other players, the incentive for joining a coalition is reduced. Proportional allocation methods that allocate cost shares on the basis of benefits gained were demonstrated to be Pareto inefficient, as they do not take into account the economies of scale encapsulated within the joint cost function. The core of game theoretic solutions, satisfying group and individual rationality, was utilized as a superior method of narrowing down the range of potential cost allocations. The imputations within the core were Pareto-efficient as they were not dominated by any other potential solutions. However, the core needed to be compacted to produce a unique optimum. Further, the core could be non-existent for any particular cost allocation game.

In a case study of areas in southern Sweden, techniques relying on allocating costs in proportion to the population were found to be inequitable (Young, Okada and Hashimoto, 1982). Further, the allocation generated by the Shapley value also failed to meet the requirements for group and individual rationality. The most desirable allocation technique was found to be the nucleolus from the perspective of equity. However, ambiguities in cost determination made the

nucleolus unusable as a solution concept. Thus, proportional allocation was actually selected as a policy.

Tisdell and Harrison (1992) modeled distribution of water licenses in Queensland, Australia using a cooperative *n*-player characteristic function game. In the absence of transaction costs and negative externalities, it is possible to achieve a Pareto-efficient reallocation of water resources without the intervention of an external regulator. However, the final allocation may be inequitable without government intervention. The allocations generated by the Shapley value and nucleolus were compared, indicating that pre-allocation of water licenses based on the market bargaining power of water users was a potential solution.

In contrast, Rosen and Sexton (1993) utilized a game theoretic approach to provide policy advice for alleviating conflict between rural water supply and municipality distribution organizations. The model developed analyzed the possible interactions between four coalitions (LO, NC, MR and ER) competing for water rights. Firstly, the coalition LO consisted of absent landowners lacking direct influence in district water supply policy. In alliance with LO was coalition NC who consisted of farmers who prefer water allocation according to land valuation. Thirdly, coalition MR consisted of farmers with a preference for resource maintaining policies. Finally, coalition ER consisted of land-renters exhibiting a preference for resource expansion. As previously, MR and ER were in alliance. The possible coalitions between the four subgroups were analyzed by determining optimal water rights allocation using the core. They determined that the application of the cooperative game theory provided an incentive for proposed water transfers, and helped to alleviate conflict between rural suppliers and distribution organizations.

Lejano and Davos (1995 and 1999) provided an expansion on the work of Rosen and Sexton (1993) by using the nucleolus concept to determine cost allocation for a wastewater reclamation problem consisting of multiple agencies. The nucleolus determined the optimal cost share distribution in terms of the rate of

return, and was monotonic with respect to total cost. That is, when the aggregate cost incurred by the grand coalition increased, the cost share for each individual in the coalition increases by the same proportion as long as the core remains non-empty. The presence of monotonicity helps to prevent a grand coalition agreement breaking apart if a water reclamation project runs over budget, as there is no incentive to leave the coalition. Their case study of a Californian wastewater reclamation project confirmed the feasibility of a cooperative game theory model in addition to the advantages of the nucleolus solution concept.

Loaiciga (2004) applied a cooperative game theoretic approach to the problem of shared ground-water extraction from an aquifer system. The aquifer system represents a common-pool resource, as the extraction of a single additional unit of water by a user prevents any other users gaining positive benefit from that unit. In addition, users with defined property rights cannot be excluded from accessing the resource. Thus, the aquifer resource is at risk of over-exploitation via a non-cooperative equilibrium, as per the 'tragedy of the commons'.

Modeling the ground-water extraction system as a *n*-player game allowed the effects of cooperative and non-cooperative behavior to be analyzed. Loaiciga (2004) determined the optimal water pumping rates for a grand coalition containing all aquifer users. Their generated unique solution maximized the net aggregate revenue received by the grand coalition. Second, the solution maximized the individual net revenue received by each aquifer user, representing a Pareto-efficient solution. In other words, none of the users had any incentive to deviate from their equilibrium strategy and extract any additional units of ground water. Any such deviation not only decreased the aggregate benefit to the coalition, but also would likely decrease the individual benefit to all of the users.

2.3.2 Experimental Studies

The literature to date on the use of experimental economics to study cooperative game theory concepts has focused on testing the predictive power of various

solution concepts. The experimental literature has been mainly focused on three and four player cooperative games due to the rapid increase in computational complexity that occurs as the number of players increases. The literature relevant to the present study is that which includes analysis of the Shapley value, nucleolus and egalitarian solution concepts.

Rapoport and Kahan (1976) tested four different models of coalition formation under laboratory conditions in a three-player super-additive cooperative game. It was found that 69% of rounds resulted in the formation of a grand coalition, with 2 player coalitions forming the remaining 31% of the total (pp. 263). Critically, the proportion of grand coalition formation increased over four repeated games, indicating that participant learning was occurring. Relevant to the present study is the comparison between the Shapley value and egalitarian solutions. The mean absolute deviation between the predicted and experimental allocations was used as a discrepancy measurement. They could not find any statistically significant differences between the relative predictive power of the Shapley value and egalitarian solutions.

Murnighan and Roth (1977), in expanding the work of Rapoport and Kahan (1976), considered the effects of communication on the allocations reached by experimental participants in a three-player cooperative game. When a single player held monopoly power over the outcome of the game, it was demonstrated that raising the level of communication increased the monopolist's payoff. In addition, they made a direct comparison between the predictive capability of the core and Shapley value, with the Shapley value offering a higher level of predictive power.

Michener, Ginsberg and Yuen (1979) performed a direct statistical comparison between the predictive power of the Shapley value, nucleolus and egalitarian allocations. They studied a four-player super-additive cooperative game incorporating side-payments. That is, the players within any ratified coalition were permitted to transfer payments between themselves in the manner of their choosing. In addition, the core of the game was non-empty, ensuring that at least

one possible allocation was within the core. The game was presented to players in payoff matrix form rather than characteristic function form. Michener, Ginsberg and Yuen (1979) found that in the first round 57.50% of allocations fell within the core, while in the second round, 62.50% of allocation were within the core (pp. 274). When the egalitarian allocation was present in the core, the number of laboratory allocations within the core was found to be greater. That is, participants exhibited significantly more difficulty forming core allocations when the egalitarian allocation fell outside the core. This study expands on this work by exploring the predictive power of these solution concepts in a 5-player game.

Michener, Ginsberg and Yuen (1979) utilized the Euclidean distance metric (Bonacich, 1979) as a discrepancy measurement for the tested solution concepts. The Euclidian (Pythagorean) distance between an observed reward vector and all of the reward vectors generated by the analyzed solution technique was determined, and the mean of all possible distances produced the goodness-of-fit score. The accuracy of the solution technique was inversely proportional to the size of the goodness-of-fit score.

If G is the set of solutions theoretically produced the analyzed technique, G contains a single unique point $\mathbf{x}$, and the observed reward vector is given by $\mathbf{o}$ then the goodness-of-fit is the Euclidean distance E as follows.

$$E = [\sum_{i=1}^{n} (o_i - x_i)^2]^{1/2} \tag{2.11}$$

A density function was utilized to calculate the goodness-of-fit when the solution set G did not contain a unique point (Bonacich, 1979; Michener et al. 1983). Let A is the magnitude of G, such that if G is a plane, A is the area under G. Thus, A is the integral over G. The inverse of A generates the density of G. As for the previous case, the goodness-of-fit E was determined by the Euclidean distance between the observed vector $\mathbf{o}$ and all vectors $\mathbf{x}$ present in G. Then E was calculated as follows.

(2.12)

$$E = \int A^{-1} [\sum_{i=1}^{n} (o_i - x_i)^2]^{1/2}. dH$$

The study demonstrated that the Shapley value produced lower discrepancy scores in comparison with the nucleolus or egalitarian allocations (Michener, Ginsberg and Yuen, 1979). That is, the Shapley value offered a significantly higher level of predictive accuracy in comparison with the compared solution concepts. However, no significant difference was found between the Shapley value and egalitarian allocations, or the nucleolus and egalitarian allocations.

Michener, Yuen and Geishenker (1980) expanded on their previous work by comparing the predictive power of the Shapley value, nucleolus and egalitarian allocations utilizing core size and non-symmetrical payoffs as primary treatment variables. The payoff equality between players in the game was used a measure of symmetry, where players could be fully interchanged in a pure symmetric game. As in the previous study (Michener Ginsberg and Yuen, 1979) experiments were performed using a four-player cooperative game with non-empty core. It was found that only 11.46% of allocations fell within the core, indicating that participants experienced difficulty in reaching core allocations for games with a small core and relatively high number of players (pp. 512). Using Bonacich's (1979) discrepancy score, the predictive power of the tested solution concepts was compared. It was found that the Shapley solution carried a higher level of predictive power than both the nucleolus and egalitarian solution. Further, the nucleolus provided a better fit for the laboratory allocations than the egalitarian solution. The authors note that the observed payoff allocations were more equalitarian when the egalitarian allocation was present within the core. Finally, all of the tested solution concepts were demonstrated to have higher predictive power under lower levels of non-symmetry in comparison with higher levels of non-symmetry.

Michener, Yuen and Sakurai (1981) provided an expansion on previous studies by comparing the predictive power of the nucleolus with core size and normal form representation as primary treatment variables. A three player super-

additive cooperative game with non-empty core was utilized for the experimental study. Out of 320 total observations, 62.50% were within the core (pp. 84). It was demonstrated that the nucleolus provided a higher level of predictive power where the core was relatively small. In addition, the column difference in the payoff matrix presented to participants was found to influence solution concept predictive power.

Previous studies by Michener et al. (1979, 1980, 1981) presented cooperative games to laboratory participants in normal matrix form. These studies determined that the column difference in the payoff matrix had a statistically significant effect on the predictive power of the tested solution concepts. Kahan and Rapoport (1981) argued that the characteristic function form offers a complete representation of a cooperative game, while there exists infinitely many normal form representations of the same cooperative game. Thus, Kahan and Rapoport (1981) provided strong evidence to support the representation of the cooperative game to participants in characteristic function form.

Michener and Yuen (1982) postulated that the core offers weak predictive power when applied to super-additive cooperative games with side-payments. The predictive power of the core was empirically compared with the Shapley value, nucleolus and egalitarian allocations using a three-player super-additive cooperative game with non-empty core. Out of 384 total observed allocations, an average of 70.05% fell within the core. Furthermore, the proportion of allocations falling within the core was greater when the core size was increased. Specifically, in large core games 79.17% of allocations fell within the core, while in small core games 60.94% of allocations were present in the core (pp. 64).

The results of Michener and Yuen's (1982) study suggested that laboratory allocations approach a more equal distribution of payoff when the egalitarian allocation lies within the core, corresponding with the findings of previous research (Michener, Ginsberg and Yuen, 1979). Using Bonacich's (1979) discrepancy metric, it was demonstrated that the core solution was less accurate than the other tested solution concepts. The core carried more predictive power

in games with a relatively small core size. However, no significant difference in accuracy between the Shapley value, nucleolus and egalitarian solutions could be determined.

The study by Michener, Potter and Sakurai (1983) provided further analysis of the predictive accuracy of the Shapley value and the nucleolus in side-payment cooperative games. The study utilized aggregate data from Michener et al. (1979, 1980, 1981, 1982) in order to compare the predictive power of the nucleolus and Shapley value. Both three and four-player side-payment cooperative games were employed in the experiments. Across the aggregated data, the nucleolus demonstrated greater mean goodness of fit scores in comparison to the Shapley value. That is, the nucleolus carried significantly less predictive power than either the Shapley value.

In an expansion of previous studies, Michener, Potter, Depies and Macheel (1984) attempted to identify classes of cooperative games in which the core offered predictive advantages. The games considered previously involved side-payments, resulting in an infinite number of pure strategies for each player. In non side-payment games, each player has a finite number of pure strategies, as coalition funds cannot be reallocated between players. Thus, players have the ability to accurate assess the set of possible outcomes before making a decision. From analysis of the experimental data, it was found that the core offers superior predictive power in finite strategy games.

In order to test the predictive power of the core in non side-payment games, Bonacich (1979) used an experimental goodness-of-fit approach to compare the predictive power of the core and set of all imputations. Two separate experiments were performed; the first involved a three-player game with three strategies; and the second, a four-player game with two strategies. The experiments were presented as a payoff matrix, and all players had complete information about the available strategies and their payoffs. Players were instructed to maximize their own payoff regardless of the payoff received by other individuals. In both experiments, face-to-face communication was

permitted and players were able to act individually or form binding coalitions with other players.

Leopold-Wildburger (1991) analyzed the effect of increasing grand coalition payoff while maintaining consistent sub-coalition values for a three-player cooperative game with side-payments. Using linear regression techniques, it was demonstrated that the nucleolus predicted a more unequal allocation between the players in comparison with laboratory results. In contrast, the Shapley value predicted a more equal distribution than the laboratory allocations.

Marchetti and Serra (2003) applied a cooperative game theory model to conflict resolution on siting terrestrial noxious facilities using voluntary market exchange. In the voluntary exchange process, a limited number of jurisdictions conduct negotiations on facility location, and then each jurisdiction is responsible for waste in their area. Due to increasing returns to scale with respect to noxious facilities, incentives existed for cooperation between multiple jurisdictions. Further, each coalition required a host location for the proposed facility. The presence of transaction costs can prevent voluntary exchange from producing an optimum distribution of facilities. In order to reduce transaction costs, Marchetti and Serra (2003) introduced an arbitrator; with the solution concept allocations providing optimal sharing strategies for the arbitrator. The predictive power of the core, nucleolus and Shapley value were compared for a three-player version of the cooperative game.

Experimental evidence was utilized to support the game theoretic predictions on distribution strategies. The optimal distributions generated by the Shapley value and the nucleolus were compared with laboratory results. It was demonstrated that 8% of coalitions formed were not the grand coalition and that 63% of payoff vectors were not part of the core (Marchetti and Serra, 2003, pp. 6). The result suggested that either calculating the core under laboratory conditions was too difficult for participants or external factors such as altruistic behavior affected participants' decisions. Further, the Shapley value was more accurate in predicting laboratory behavior and producing an efficient siting location, in

comparison to the nucleolus. Therefore, the Shapley value solution was recommended for the arbitrator in order to resolve conflict in siting noxious facilities.

Bolton, Chatterjee and McGinn (2003) investigated the effect of communication in coalition allocations using a 3-player cooperative game with side-payments. Three treatments were considered; unconstrained communication, in which participants could choose to communicate privately or publicly with any other player; public, in which messages were broadcast to all players; and player-control, in which all messages had to pass through or be generated by a single player. It was demonstrated that the proportion of grand coalitions formed was greater when the two weakest players held communication privileges (between 35% and 80% of total formed coalitions being the grand) (pp. 588). Further, unconstrained communication resulted in the lowest rates of grand coalition formation (10% of total formed coalitions being the grand) (pp. 588). For all treatments, the rate of grand coalition formation increased as the number of experiment rounds increased, indicating participant learning was a factor in forming the grand coalition.

Tutic, Pfau and Casajus (2011) analyzed the predictive power of the nucleolus using a cooperative glove game. In the glove game, participants were classified into left or right glove holders. Any two players could form a coalition, with the coalition receiving the maximum possible payoff if it contained both a left and right glove. Thus, the grand coalition consisted of two players, with one holding a right glove and one holding a left glove. The predictive power of the nucleolus was analyzed for the glove game experiments. However, it was demonstrated that when data was aggregated across all experiments, there was no significant difference between the accuracy of the nucleolus and the other tested solution concepts. However, the goodness-of-fit scores of the nucleolus were shown to decline as the number of repeated interactions increased, indicating that its predictive power increased with participant learning.

Beimborn (2013) analyzed how the costs and benefits generated by a coalition

should be allocated to maximize coalition stability. Cooperative game theory was applied to cooperative sourcing agreements, in which several players combine resources in order to reduce overall costs via economies of scale. The predictive power of proportional cost allocation, in which members of a coalition receive cost shares on the basis of individual cost, was compared with allocation via the Shapley value. In addition, coalition stability under the two allocation techniques was analyzed.

The experimental cooperative game consisted of three players each with private information regarding cost structures. Players were permitted to communicate face-to-face and either become part of a coalition or act independently. Beimborn (2013) concluded that; firstly, participants rarely considered coalition stability in their decision-making; secondly, the Shapley value offered the greatest predictive power among the allocation methods analyzed; and thirdly, proportional cost allocation evolved when information regarding cost structures was partially revealed. In other words, in the presence of asymmetric information, the probability of coalition instability increases as allocations approach that of the Shapley value. Reducing the information asymmetry increases the chance of a stable coalition.

2.4 Rationale for the Present Study

This study builds on the work of Michener et al. (1979, 1982) and Beimborn (2013) by competitively testing the predictive power of the Shapley value, nucleolus and egalitarian allocations in a 5-player game, using an experimental economics approach. In order to provide a realistic representation of conservation negotiations on the landscape scale, the number of players in the cooperative game was increased in the present study in comparison with previous work by Michener et al. (1979, 1982) and Beimborn (2013). In corollary with studies by Michener et al. (1979, 1980, 1981, 1982, 1983, 1984), the cooperative game in the present study allows coalition participants to freely allocate their aggregate payoff. That is, side-payments were permitted. Further, the cooperative game was presented to participants in characteristic function

form rather than normal matrix form[1] in order to avoid influencing the predictive power of the tested solution concepts (Kahan and Rapoport, 1981).

The experimental studies discussed in this chapter highlight that the difficulty for participants in reaching a core rises as the number of players in the game increases (Michener et al. 1980). Given that the present study expands on previous work by increasing the number of players in the game, the core size was maximized and the equal allocation was placed within the core. As previous studies (Murnighan and Roth, 1977; Bolton et al. 2003) determined that unrestricted communication reduced the incidence of allocations within the core, no communication as permitted between participants in the present study. Finally, the core was not tested as a solution concept as it was demonstrated to offer inferior predictive capability to the Shapley value, nucleolus or egalitarian allocation.

Therefore, based on Michener et al. (1979, 1982) and Beimborn (2013), the following null hypotheses for the present study can be devised.

- *Hypothesis 1: There is no significant difference between theoretical solution concept payoff allocations and laboratory allocations.*
- *Hypothesis 2: There is no significant difference between the mean discrepancy scores of the Shapley value, nucleolus and egalitarian allocations.*

2.5 Summary

[1] In normal form, player strategies are represented in a decision matrix where each cell displays the payoff vector generated by applying the corresponding strategy.

This chapter has outlined the principles of cooperative game theory, application to natural resource problems and studies in experimental economics. The presented literature provided justification for an experimental study designed to test the accuracy of the Shapley value, nucleolus and egalitarian solution concepts, with respect to landscape-scale conservation negotiations. Chapter 3 will build on this chapter by detailing the methodology and process utilized to parameterize the cooperative game, and the experimental design for comparing the predictive power of the tested solution concepts under laboratory conditions.

Chapter 3

Methodology

3.1 Outline

This chapter builds on Chapter 2 by providing a detailed description of the methodology utilized in this study to competitively test the predictive power of the Shapley value, nucleolus and egalitarian solution concepts. Chapter 3 describes the algorithms and GAMS computer code used to calculate the core boundaries, as well as the Shapley value, nucleolus and egalitarian allocation for any cooperative game. The application of these computer procedures were then applied to parameterize a five-player cooperative game that represents a potential real-world interaction between landowners in a given landscape. Finally, the parameterized five-player game was applied to a series of laboratory experiments with the objective of comparing the allocations selected by experimental subjects with the allocations predicted by the aforementioned solution concepts.

3.2 Procedures for Determining the Core, Shapley Value, Nucleolus and Egalitarian Allocation

The core, Shapley value, nucleolus and egalitarian solution for a generic cooperative game were determined using linear programming. Procedures written in GAMS©[2] (General Algebraic Modeling System) were utilized to solve all linear programming problems. The core, Shapley value, nucleolus and egalitarian allocation were represented as separate procedures within GAMS. The LP (linear programming) solver was used to solve each procedure. The procedures and LP algorithms are described in the following sections. GAMS code for all procedures are detailed in Appendix A.

[2] The GAMS Development Corporation, Washington DC, http://www.gams.com/contacti.htm

3.2.1 Core Existence and Maximum Limits

The procedure for determining the existence of a core for the parameterized cooperative game is shown in Program 1 in Appendix A. The procedure returns a single payoff vector within the core if the core exists and is stable. If the core does not exist, the procedure returns an unfeasible solution.

The procedure consisted of the following steps:

- Store the values associated with each coalition according to the characteristic function.
- Initialize the core constraints for each sub-coalition and the grand coalition.
- Maximize the aggregate payoff subject to the core constraints
- Return a vector containing an allocation within the core, therefore confirming a non-empty core.

The algorithm followed in the GAMS program is shown in Pseudo-code 3.1.

Pseudo-code 3.1. Existence of a Core.

[Algorithm: **x**]	
x = **0**;	(1). Set the initial payoff of each player to be zero.
for $\forall\, S\ \subseteq N$ **do** S = V(S); **end for;**	(2). For each coalition in the game, store the value of that coalition in *S*, according to the characteristic function *V(S)*.
for $\forall\, S\ \subset N$ **do** $\sum_{i \in S} x_i\ \geq V(S)$; **end for;**	(3). For each coalition *S*, enforce the constraint that the sum of individual player payoffs must be greater than or equal to the value *V(S)*.
$\sum_{i \in S} x_i = V(N)$;	(4). Enforce the grand coalition constraint; the sum of player payoffs must be equal to the value of the grand coalition *V(N)*.
$\max z = \sum_{i \in S} x_i$; **x** = z;	(5). Maximize the sum of individual player payoffs subject to the defined constraints and return the reward vector **x**.

return x;

The procedures shown in Programs 2 to 6 in the Appendix were utilized to determine the maximum limits of the core for each player. Each procedure returns a single value representing the maximum possible payoff for the associated player that is still within the core.

The procedure consisted of the following steps:

- Store the values associated with each coalition according to the characteristic function.
- Initialize the core constraints for each sub-coalition and the grand coalition.
- For each player, maximize his or her received payoff subject to the core constraints.
- Return a vector containing the core limit allocation for each player.

The algorithm used to calculate the core limits is shown in Pseudo-code 3.2.

Pseudo-code 3.2. Maximum core limits.

[Algorithm: **x**]

Repeat steps (1) to (4) in Pseudo-code 3.1.

for $\forall\ i\ \in n$ **do** max $z = x_i$; **end for;** **return x**;	(5). For each player i, maximize that player's individual payoff x_i subject to the defined constraints and return the reward vector **x**.

3.2.2 Shapley Value

Once the existence and stability of the core was confirmed, the Shapley allocation was determined using the procedure in Program 7 in Appendix A. The procedure

calculated a single payoff vector corresponding to the Shapley allocation. The following programming pseudo-code represents the flow of control within the Shapley algorithm.

The procedure consisted of the following steps:

- Store the values associated with each coalition according to the characteristic function.
- Store the number of players required for each coalition as a parameter.
- For each player in the game, calculate their allocation according to the Shapley formula.
- Return a vector containing the allocation to each player.

The algorithm followed in the GAMS program is shown in Pseudo-code 3.3.

Pseudo-code 3.3. Calculation of the Shapley value.

[Algorithm: **x**]

Repeat steps (1) and (2) in Pseudo-code 3.1.

for $\forall S \subseteq N$ **do**

n = F(S);

end for;

(3). For each coalition in the game, store the number of players required to complete that coalition in the parameter *n*.

for $\forall i \in \{1, 2, 3, 4, 5\}$ **do**

$x_i = \sum_{all\ S\ s.t\ i \in S} \frac{(|S|-1)!(n-|S|)!}{n!} [v(S) - v(S - \{i\})];$

end for;

x = z;

return x;

(4). For each player in the game, calculate their payoff according to the Shapley formula. Return the payoff vector **x** containing the individual payoff of each player.

3.2.3 Nucleolus

The nucleolus of the cooperative game was calculated after the existence of the core was confirmed. Following the procedure described by Tisdell and Harrison (1992), the first iteration of the nucleolus algorithm was performed by minimizing the excess, given by Equation (9), subject to the constraints in

Equations (10), (11) and (12). The first iteration was performed using a linear programming approach as demonstrated by the procedure in Program 8 of Appendix A. If a unique solution for the linear program was reached after the first iteration of the procedure, the algorithm concluded and the allocation found was equal to the nucleolus. A unique solution occurred when the nucleolus constraints were satisfied with a single payoff vector. However, if the procedure did not return a unique solution after the first iteration, all constraints with excesses equal to the minimum excess were excluded. This was achieved by equalizing the inequality of these constraints and substituting the minimum excess value into each equation. The linear programming procedure was then run again as a further iteration until a unique solution was reached.

The procedure consisted of the following steps:

- Store the values associated with each coalition according to the characteristic function.
- Initialize nucleolus constraints for each sub-coalition and the grand coalition.
- Minimize the excess subject to the defined nucleolus constraints.
- Remove all constraints with excesses equal to the minimum excess.
- Repeat the iteration while the minimum excess is less than zero.
- Return a vector containing the allocation to each player according to the nucleolus.

The algorithm is represented in Pseudo-code 3.4.

Pseudo-code 3.4. Calculation of the nucleolus.

[Algorithm: **x**]

Repeat step (1) in Pseudo-code 3.1.

while e $\geq$ 0 **do** — (2). Repeat nucleolus calculation until the excess e is equal to zero. Once e is equal to zero, an exact solution has been found.

Repeat step (2) in Pseudo-code 1.

for $\forall S \subset N$ **do** — (3). For each coalition, enforce the constraint that the sum of individual player payoff and the excess must be greater than or equal to the value of the coalition *V(S)*.

$\sum_{i \in S} x_i + e \geq V(S)$;

end for;

Repeat step (4) in Pseudo-code 1.

min $z = \mathrm{e}$;

end while;

(5). Minimize the excess e subject to the defined constraints.

return x;

(6). Return the vector **x** containing the payoffs of each individual player.

3.2.4 Egalitarian Allocation

The egalitarian allocation provides each player with an equal allocation regardless of the value of his or her individual coalition. Although this solution concept is mathematically simplistic, its implementation has been demonstrated to provide accurate predictions of subject behavior (Michener, Yuen and Geishenker, 1980). The payoff to each player is found by dividing the value of the grand coalition by the total number of players in the game. The algorithm is shown in Pseudo-code 5 as follows.

[Algorithm: **x**]

Repeat step (1) in Pseudo-code 3.1.

n = 5;

(2). Set *n* to be the number of players.

for $\forall\, i \in \{1, 2, 3, 4, 5\}$ **do**

$x_i = V(N)/n$;

(3). For each player in the game, calculate their payoff by dividing the value of the grand coalition *V(N)* by the number of players *n*.

end for;

return x;

(4). Return the vector **x** containing the payoffs of each individual player.

Using the algorithmic procedures for the core, appropriate values were selected for each coalition in order to generate the player's profile. The players' profiles were designed to be a realistic representation of conservation negotiations between landowners in a region. The following section describes the parameterization of the players' profiles.

3.3 Generation of the Players' Profiles

As previously mentioned, the laboratory experiments have been modeled ing terms of a hypothetical conservation scenario. Suppose there are a group of private landowners in a region. Across the various farms in the region, the relative land area and productivity varies. Therefore, each landowner receives a different payoff from farming his or her land and acting independently. Alternatively, two or more landowners have the option to forfeit their individual payment and provide their land for the purposes of forming a bioregion. In this case, the coalition will be compensated for the conservation value of the bioregion. The more landowners forming such a coalition, the greater the relative value of the bioregion. Further, unique environmental characteristics within each area of land can contribute to the value of a bioregion. For example, two adjoining properties may connect a river habitat, thereby adding to the conservation value of a region by the inclusion of those two properties. In this study, the players profile was designed to be a hypothetical representation of this scenario.

The players profile for the cooperative game was generated using a sensitivity analysis. The purpose of this sensitivity analysis was to determine appropriate values for each possible coalition according to defined selection criteria. That is, the characteristic function for the game was produced from the sensitivity analysis results. The selection criteria chosen for this study were the number of players; core existence and stability; the magnitude of benefits for the formation of higher coalitions; and maximum distance between the reward vectors generated by the Shapley value and nucleolus solutions.

Previous laboratory studies have focused on three and four-player cooperative games in characteristic function form (Kahan and Rapoport, 1975, 1981; Michener et al, 1980, 1981, 1982, 1983; Marchetti and Serra, 2003; Tutic et al, 2011; Beimborn, 2013). In order to provide a more realistic representation of the potential interactions expected between a group of landowners in a landscape, a compromise position was sought between the number of players

and the resulting complexity of the game. Therefore, in this study the number of players was increased to five, permitting application to complex landscape conservation problems. A game of six or more players was rejected due to the calculation difficulties imposed on laboratory participants by the high number of potential coalitions. For example, a five-player game presents each player with 31 potential coalitions, while a six-player game increases the number of coalitions to 63 which was deemed to be intractable for the laboratory.

To isolate the predictive power of the tested solution concepts, this study has identified the presence of a non-empty, stable core as a crucial parameterization criterion. Previous studies have demonstrated that the presence of an empty core influences the decisions made by players under experimental conditions (Michener et al, 1981, 1989). This observation is due to changes in the strategic decision faced by players in relation to whether the core is empty or non-empty (Shapley and Shubik, 1973; Young, 1979). A series of experiments performed by Kahan and Rapoport (1975) were conducted on the basis of a game with an empty core. However, Michener et al. (1981) demonstrated that the number of coalitions formed under experimental conditions reduced when the core was empty. Further, they postulated that the presence of an empty core reduced the incentive for the formation of higher coalitions, particularly the grand coalition. Michener et al. (1981) argued that a non-empty core externally influenced the performance of solution concepts, potentially invalidating the experimental results. For the parameterization of this 5-player game, the values for each sub-coalition were therefore adjusted until a stable, non-empty core was found.

In the present study, it was identified that lucid benefits were required to strongly incentivize the formation of higher coalitions, in addition to satisfying the super-additive axioms. That is, the formation of a coalition between players needed to offer a clearly measureable gain in payoff compared to acting individually or in smaller coalitions. This parameterization criterion was critical to reduce the complexity of calculations for laboratory participants, allowing reasonable negotiation time within each experiment round. With this criterion in consideration, the values of sub-coalitions were again adjusted to incorporate

lucid benefits while maintaining a non-empty core. The final values of each potential coalition used in the experiments are shown in Table 3.1, and represent the 5-player cooperative game in characteristic function form.

Table 3.1. Values of each coalition.

Coalition	Payoff	Coalition	Payoff
{0}	0	{1,2,3}	94
{1}	35	{1,2,4}	89
{2}	25	{1,2,5}	106
{3}	20	{1,3,4}	98
{4}	15	{1,3,5}	97
{5}	30	{1,4,5}	90
{1,2}	65	{2,3,4}	90
{1,3}	59	{2,3,5}	91
{1,4}	53	{2,4,5}	78
{1,5}	70	{3,4,5}	85
{2,3}	51	{1,2,3,4}	130
{2,4}	42	{1,2,3,5}	141
{2,5}	61	{1,2,4,5}	131
{3,4}	54	{1,3,4,5}	136
{3,5}	57	{2,3,4,5}	132
{4,5}	49	{1,2,3,4,5}	180

In this study it was postulated that the distance between the Shapley value and nucleolus allocations within the core should be maximized in order to allow statistical inference on the relative predictive power of the tested solution concepts. Further, it was hypothesized that as the magnitude of the grand coalition increased, the distance between the Shapley value and nucleolus would proportionally increase. Therefore, it was required to determine a range of possible values for the grand coalition, and measure the distance between the

Shapley value and nucleolus at each point. This was achieved by firstly minimizing the value of the grand coalition while maintaining a stable, non-empty core. The minimum possible grand coalition value was demonstrated to be 170. The value of the grand coalition was then incremented by 5 until a value of 220 was reached, and the Euclidean distance between the Shapley value and nucleolus solutions was measured at each iteration. In previous studies, the Euclidean distance was utilized as a discrepancy metric between separate reward vectors (Michener et al, 1980, 1981, 1982, 1983). The results of this parameterization are shown in Figure 3.1. The maximum Euclidean distance was found to be at 180. Grand coalitions with value above 220 were not considered in the analysis due to providing too great an incentive for participants to enter the grand coalition.

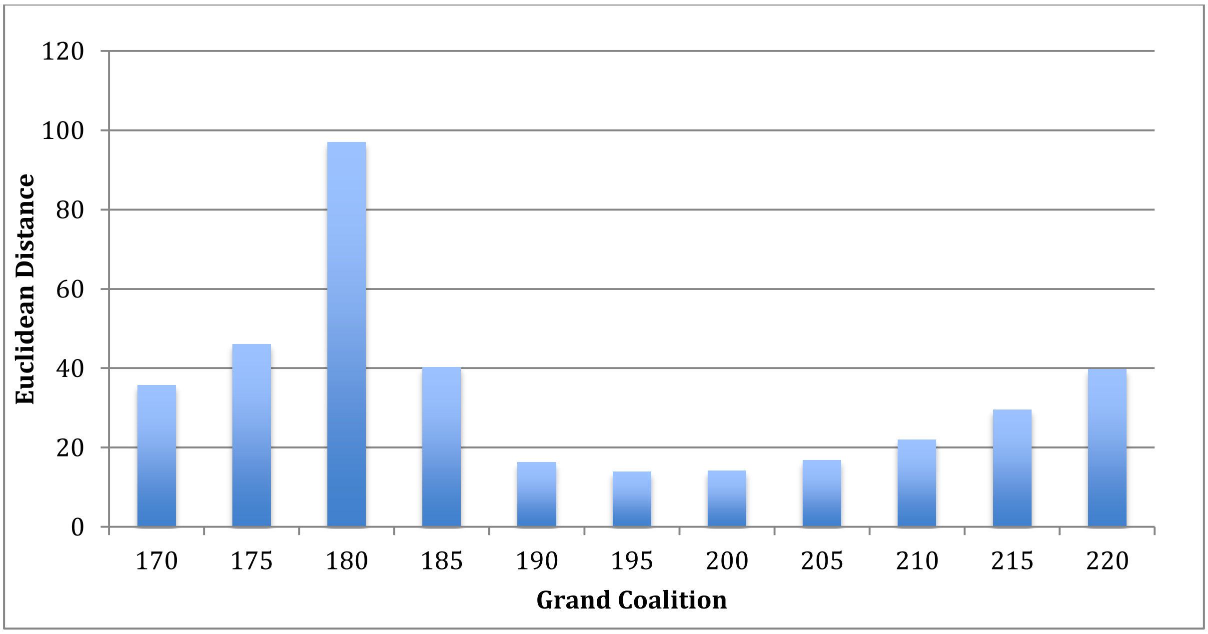

Figure 3.1. Discrepancy between Shapley value and nucleolus solutions as a function of grand coalition value

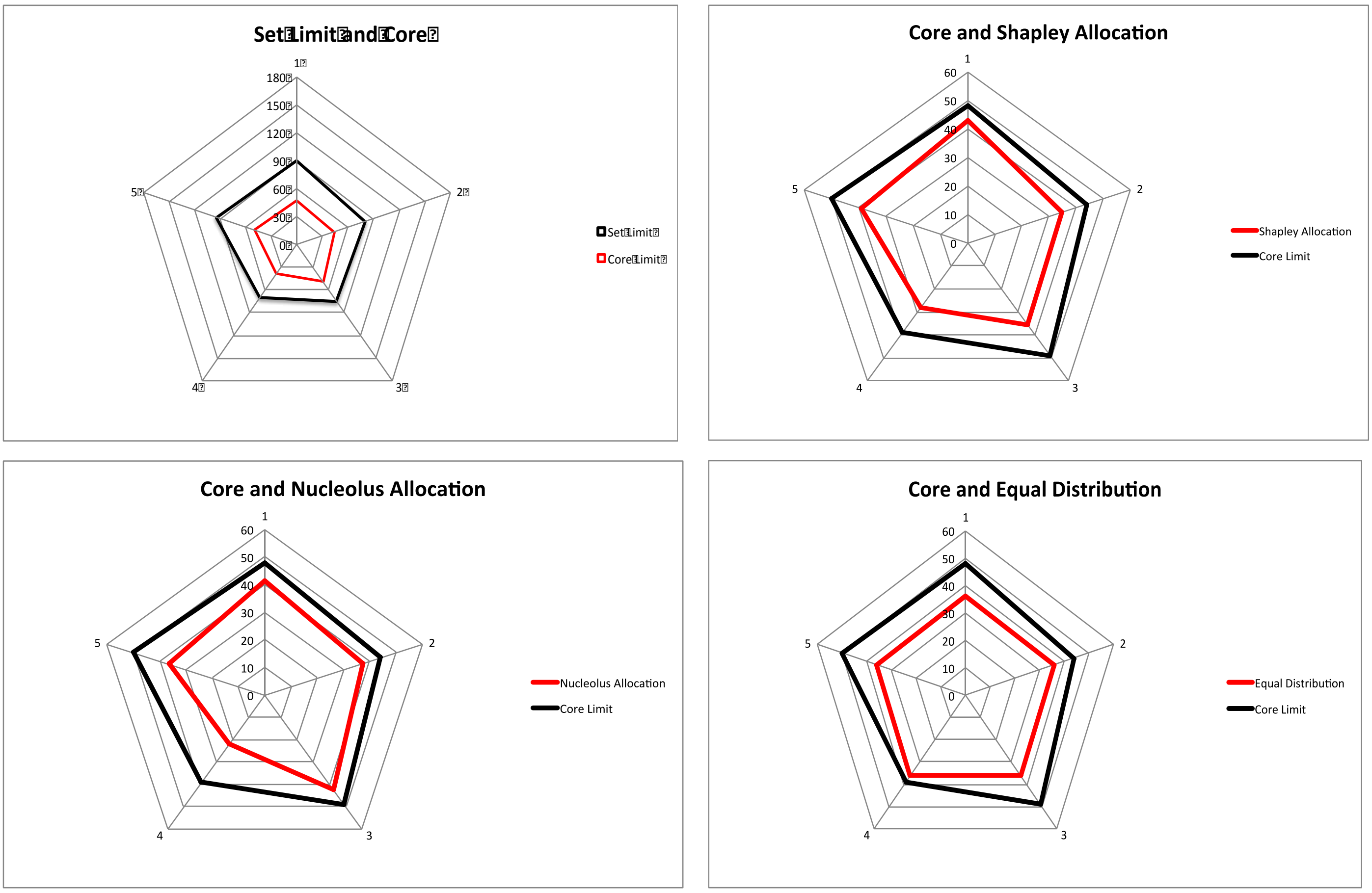

Figure 3.2. Graphical representation of the core and the allocations determined by each tested solution concept.

The limits of the core boundaries are shown in the upper left hand quadrant of Figure 3.2. This displays the core within the maximum attainable payoffs for each player, regardless of the limitations imposed by the core. That is, each player's maximum attainable payoff is that which is received if all other players negotiate an allocation equal to the value of their individual coalitions.

The remaining three quadrants graphically depict the allocations predicted by the Shapley value, nucleolus and egalitarian solutions respectively. These graphs demonstrate that all three tested solution concepts fall within the core. In comparison with the egalitarian allocation, both the Shapley value and nucleolus provide a lower reward to Player 4. Conversely, Player 1 earns a higher payoff under the Shapley value and nucleolus than under an egalitarian allocation.

Finally, Player 3 receives a higher payoff under the nucleolus by comparison with the other solution concepts.

3.4 Predictions

The reward vectors for each solution concept shown in Table 3.2 were used as the basis for predictions of subject behavior. Following Michener et al. (1980), the validity of the predictions rests on an assumption of players having an individualistic motivational orientation, that is, seeking personal gain without regard for the payoffs received by other players. From Table 3.2, it can be seen that player 1 should accept the Shapley allocation in preference to the nucleolus and egalitarian solutions, as it offers the highest payoff. Similarly, Players 2 and 3 should opt for the nucleolus allocation; Player 4 should approach the egalitarian allocation; and Player 5 should choose the Shapley allocation.

Table 3.2. Summary of payoff vectors from tested solution concepts for the final parameterized game for each player in the game.

	1	2	3	4	5
Shapley Allocation	42.8	34.8	35.6	27.8	39
Nucleolus Allocation	41.5	37.5	42.5	22	36.5
Egalitarian Allocation	36	36	36	36	36
Individual Payoff	35	25	20	15	30
Loss incurred by second best	3.71%	6.00%	32.50%	54.67%	8.33%
Loss incurred by third best	19.43%	10.80%	34.50%	93.33%	10.00%

Therefore, none of the tested solution concepts are preferred by a majority of the players. When the second best allocation for each player is considered, the nucleolus produces the preferred allocation for the majority of the players. Further, allocating according to the nucleolus solution results in a relatively small loss for Players 1 and 5. However, Player 4 has a relatively high level of dissatisfaction when the nucleolus allocation is selected. In this study, it is

therefore hypothesized that Player 4 will negotiate strongly for an egalitarian solution. Under these conditions, Players 2 and 3 will receive their second best payoff, while Players 1 and 5 will receive their third best payoff. However, Players 1 and 5 are more likely to accept the egalitarian solution than Player 4 is to accept the Shapley value or nucleolus allocations. Therefore, it can be hypothesized that a negotiated allocation will approach the egalitarian solution.

3.5 Experimental Procedures

The following section describes the design and process of the experiments performed to test the predictive accuracy of the Shapley value, nucleolus and egalitarian solution concepts.

3.5.1 Experimental Design

The laboratory experiments were conducted using a five-player game, using the coalition values presented in Table 1. Therefore, the selected game was super-additive and contained a non-empty core (Michener, 1980). Previous studies have been divided on the form of cooperative games presented in the laboratory, with representations in either characteristic form (Kahan and Rapoport, 1976, 1981) and normal matrix form (Michener et al, 1980, 1981, 1982). The characteristic function form has been demonstrated to be a complete strategic representation of the choices available to each player (Kahan and Rapoport, 1981). Further, Kahan and Rapoport (1981) argued that differing normal form representations of a single game in characteristic function form affected the frequency of grand coalition formation and the proportion of allocations falling within the core. Thus, the game was presented to players in characteristic function form in order to analyze the predictive power of solution concepts.

3.5.2 Subjects and Procedures

The series of experiments were conducted at the University of Tasmania using custom-designed computer software. Laboratory subjects were 50 volunteer undergraduate and postgraduate students at the University of Tasmania, comprising 10 independent five-person groups. The laboratory advertised for experiment positions throughout the University semester. Subjects were permitted to register expressions of interest using an online booking system. The system randomly selected subjects for experiments on the basis of producing unique sets. All experiments were conducted using a customized module developed for TESS© (The Experimental Systems Software[3]). On average, each experiment was completed in one hour and fifteen minutes.

Each experiment session consisted of exactly five participants. Each participant was randomly and confidentially allocated a player number (1, 2, 3, 4, 5). At the start of each experiment, the participants were told to read a set of instructions on a computer terminal. Once they had finished the instructions, they were required to complete a Microsoft Excel© based quiz with the aim of testing their understanding of the instruction set. A complete set of the instructions and quiz are presented in Appendix B. The experiment supervisor was present in the laboratory room, and participants were permitted to ask questions of the supervisor. However, no verbal communication between participants was allowed in the laboratory room for the duration of the experiment. All interactions between participants during the experiment were limited to directly through the experiment software.

The experiment scenario consisted of 12 independent rounds. In each round, each participant had the opportunity to enter any of his or her available coalitions with other anonymous participants. Only the coalitions possible for each player to join were visible on the experiment screen. To enter a coalition with one or more other participants, a player was required to enter the coalition

[3] The Experimental Economics Laboratory, University of Tasmania, http://www.utas.edu.au/economics-finance/experimental-economics.

ID number and his or her offer price into the experiment software. A valid offer price was required to be greater than or equal to the value of the player's singleton coalition, and less than or equal to the value the appropriate coalition. The coalition ID number was comprised of the players required to fulfil the coalition. Players were not required to form a coalition during each round, and were endowed with an "individual income" equal to the value of their singleton coalition. Further, each player was permitted to withdraw or modify a standing offer at any time during the round. However, each player was allowed to have only one standing offer on a coalition at any time during a round.

The status of offers on available coalitions were displayed in a table on the computer screen of each participant for the duration of each round. Each round had a duration of 180 seconds. On the conclusion of each round, players were paid the value of any standing offers on a *valid* coalition in experimental dollars. A valid coalition required offers from all of the requisite players, and the aggregate value of those offers had to be less than or equal to the value of the coalition. Any player that had no standing offers on a valid coalition were paid the value of their individual income in experimental dollars. On the conclusion of each round, players could view their income in experimental and actual dollars, in addition to information on which coalition they successfully joined (if any). The calculation of actual income varied depending on the player. The metric utilized to determine the income of each player is shown in Table 3.3. To calculate a player's actual income, the experimental income of the player multiplied by the appropriate multiplier was added to the appropriate constant.

Table 3.3. Metric used to determine the actual income of each player.

Player	1	2	3	4	5
Multiplier	0.244	0.167	0.109	0.132	0.158
Constant	-8.373	-4.008	-2.013	-1.813	-4.573

On the conclusion of each session, participants were confidentially paid their income in cash. Income from the experiment ranged between $15 and $45, with an average of $30.

3.6 Summary

This chapter has defined the methodology utilized to parameterize a five-player cooperative game that represents real-world negotiations between landowners in a landscape. The core of the game and reward allocations using the Shapley value, nucleolus and egalitarian solutions have been determined. Further, the experimental design used to compare the relative predictive power of the three tested solution concepts has been detailed. Chapter 4 will expand by providing statistical analysis of the collected laboratory data to judge the predictive ability of the Shapley value, nucleolus and egalitarian solution concepts.

Chapter 4

Results

4.1. Introduction

Chapter 4 provides a summary of the experimental and simulation results. The chapter begins with the comparative predictive performance of the Shapley value, nucleolus and egalitarian solutions for the tested five-player cooperative game. Analysis of the laboratory results was presented following a three-tiered approach: (1) descriptive statistics on the proportion of allocations formed within the core; (2) statistical inference on the discrepancies between the solution concept predictions and experimental allocations for aggregated data across all sessions; and (3) statistical inference on each individual round to account for changes in participant behavior due to learning across rounds.

4.2. Descriptive Statistics

Following Beimborn (2013), the proportion of successful allocations within the core was determined. Table 4.1 displays the frequency by which observed allocations fell within the core of the game. Out of 120 observed rounds, only 44% of rounds resulted in an allocation within the core. The grand coalition was reached in 48% of rounds, with 4% of rounds resulting in a grand coalition outside of the core. These formed grand coalitions were outside of the core as the aggregate payoff allocation was less than the value of the grand coalition. In 19% of rounds, participants were only able to reach their singleton coalitions. In addition, 33% of rounds resulted in partial coalitions (not including the grand coalition). This indicates that participants experienced difficulty in consistently reaching the grand coalition.

Table 4.1. Proportions of Coalition Formation.

Coalition		Frequency	% of Total Coalitions
Singleton		23	19%
Partial		39	33%
Grand	Non-Core	5	4%
	Core	53	44%
Total		120	100%

The absolute mean percentage deviations of each player's payoff from the predicted values was used as an initial indication of predictive power (Beimborn, 2013). This, in combination with the proportion of laboratory allocations that fell within the core, was used to establish the level of difficulty experienced by participants in reaching the grand coalition.

Table 4.2 displays the mean percentage deviations of laboratory allocations from the solution concept predictions for all formed coalitions (including singletons). The egalitarian allocation provided the lowest mean percentage deviation for the payoffs of players 1, 2, 4, and 5, while the Shapley value provided the smallest deviation for the payoff of player 3.

Table 4.2. Mean percentage deviations for all coalitions

Mean % Deviation (All Coalitions)	Player				
	1	2	3	4	5
Shapley	15.52	13.96	18.93	29.91	14.62
Nucleolus	12.89	16.53	31.18	46.70	9.38
Egalitarian	2.99	13.80	18.98	23.03	8.29

Table 4.3 displays the mean percentage deviations from solution concept predictions when only allocations within the core were considered. The egalitarian allocation produced smaller mean percentage deviations in comparison with the Shapley value and nucleolus for players 1, 2, 3 and 4. However, the nucleolus experienced the least deviation for the payoff of player 5.

Table 4.3. Mean percentage deviations from solution concepts, allocations within the core.

Mean % Deviation (Core)	Player				
	1	2	3	4	5
Shapley Value	14.55	4.65	2.70	27.93	8.89
Nucleolus	11.84	4.28	16.55	61.76	2.49
Egalitarian	1.92	1.82	1.96	1.92	3.59

Table 4.4 displays the mean percentage deviations from solution concepts predictions when singleton and sub-coalitions were considered. The egalitarian allocation provided the lowest mean percentage deviation scores for the individual payoffs of players 1 and 5, while the Shapley value produced the lowest deviation for the payoffs of 2, 3, and 4.

Table 4.4. Mean percentage deviations from solution concepts, allocations outside the core.

Mean % Deviation (Non-Core)	Player				
	1	2	3	4	5
Shapley Value	16.42	22.67	34.11	31.76	19.98
Nucleolus	13.87	28.00	44.86	32.62	14.80
Egalitarian	3.99	25.00	34.90	42.79	13.71

4.3 Aggregate Payoff on a Round-by Round Basis

The aggregate payoff earned in each experiment round was utilized to analyze the difficulty experienced by participants in reaching the grand coalition. Figure 4.1 displays the aggregate payoff as a function of round number for each individual session. The grand coalition was consistently reached in the majority of rounds in sessions 168, 172, 176 and 177. Further, an increasing trend towards the grand coalition payoff was demonstrated in sessions 179, 183 and 185. This increasing trend provided some empirical evidence to suggest participant learning as the experiment progressed. For the remaining sessions, the participants failed to consistently form the grand coalition in the majority of rounds.

Wilcoxon's rank sum test was used to determine which sessions achieved the grand coalition in the majority of rounds. The aggregate player payoff for each round was statistically compared with the value of the grand coalition. For sessions 168, 172, 176 and 177, no significant difference was found between aggregate player payoffs and the grand coalition value ($p > 0.05$). The aggregate player playoff was significantly less than the grand coalition value in sessions 170, 174, 179, 181, 183 and 185 ($p < 0.05$). Therefore, participants failed to reach the grand coalition in 60% of sessions, necessitating separate analysis of solution concept performance for cases where only sub-coalitions were reached.

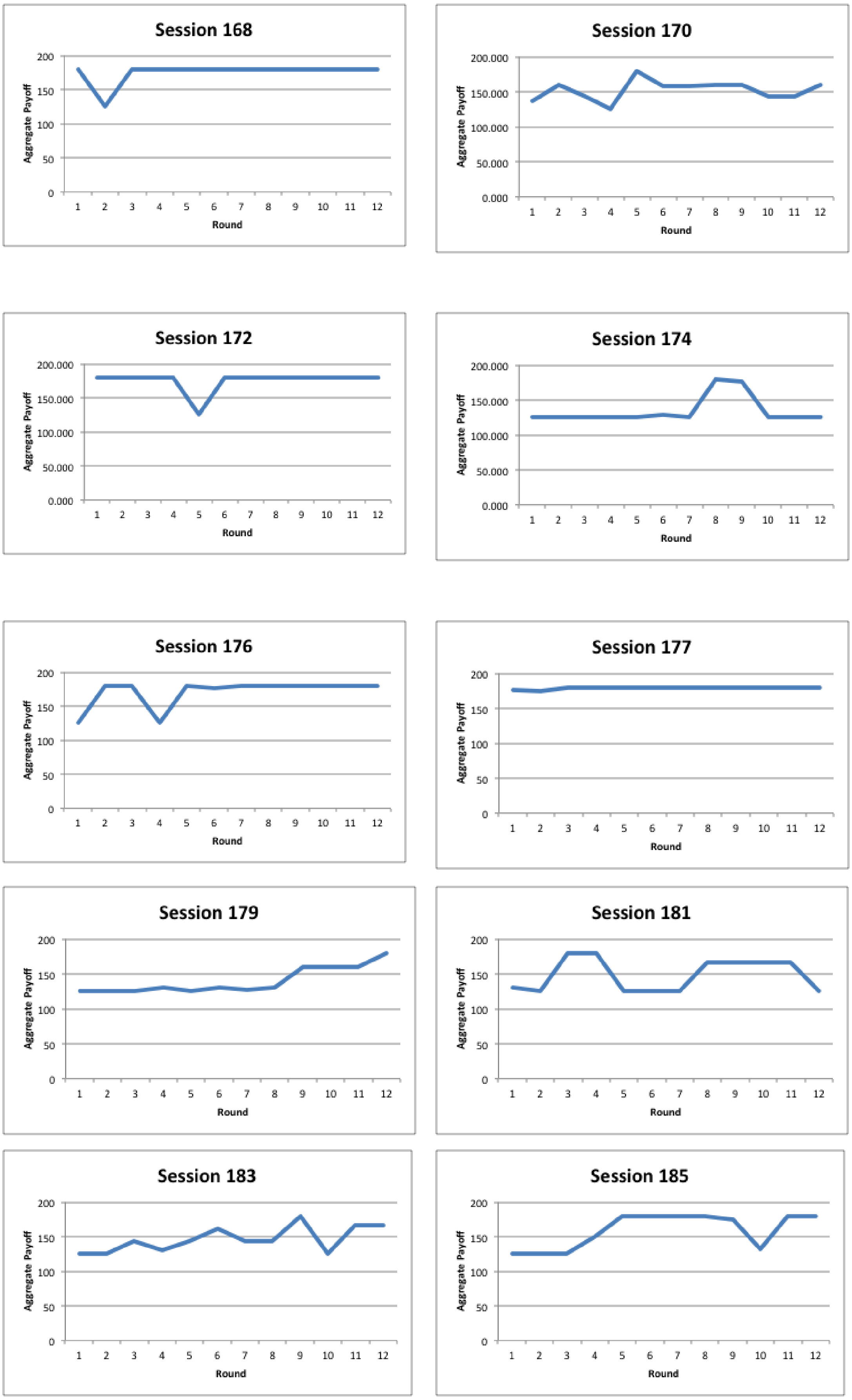

Figure 4.1. Aggregate player payoff as a function of round number.

4.4 Statistical Inference on Solution Concept Predictive Power

The primary objective of this study was to compare the predictive power of the Shapley value, nucleolus and egalitarian solution concepts. The Euclidean distance, or discrepancy score, between the predicted and experimental allocation was utilized as a quantitative metric for solution concept accuracy, with a lower discrepancy score indicating a higher degree of accuracy. This discrepancy score was calculated for every group on each round. The mean discrepancy scores for each round across 10 sessions are shown in Figure 4.2.

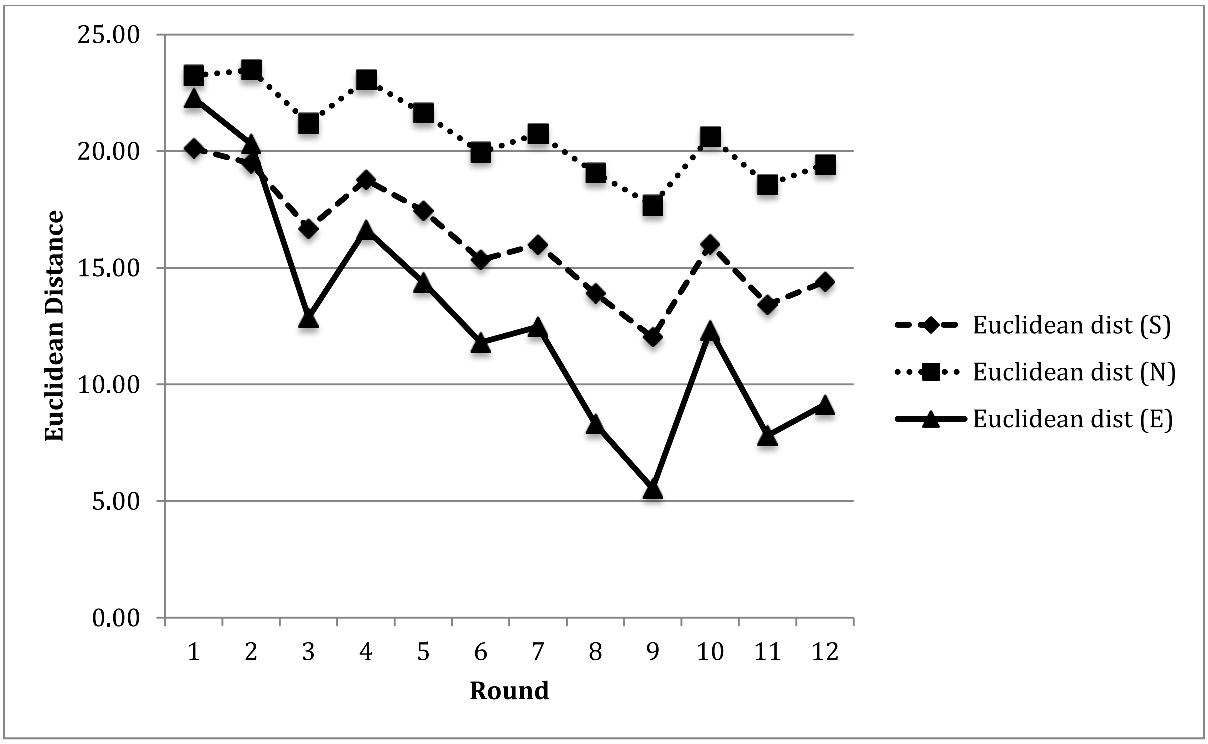

Figure 4.2. Mean Euclidean distances between observed results and Shapley value (S), nucleolus (N) and egalitarian (E) predictions.

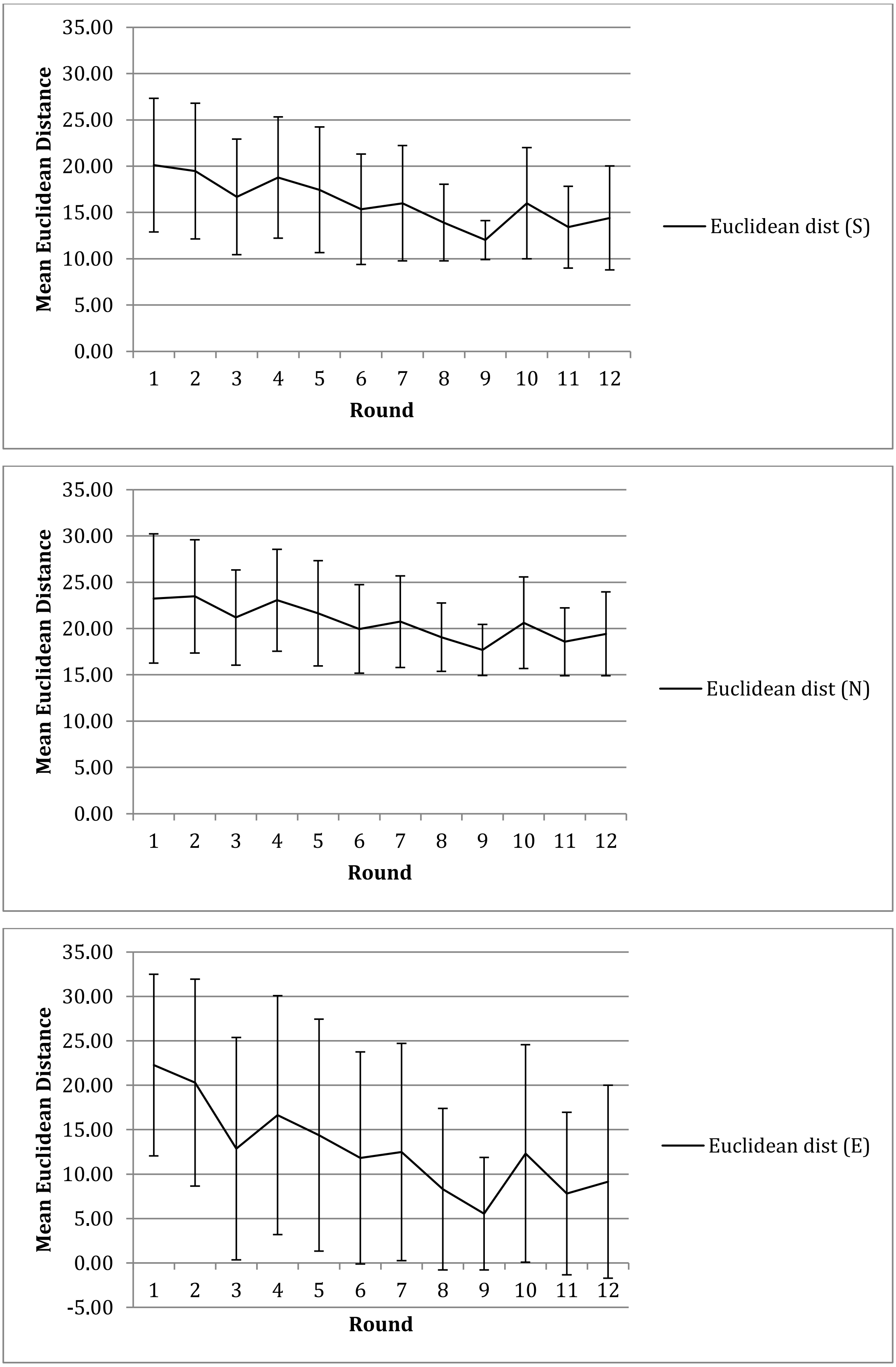

Figure 4.3. Mean Euclidean distances for Shapley value (S), nucleolus (N) and egalitarian (E) predictions (including standard errors).

Figure 4.3 shows that the mean discrepancy between the Shapley value, nucleolus and egalitarian solutions was relatively small within the first and second rounds. As the number of repeated rounds increased, the average discrepancy for each solution concept displayed a decreasing trend, declining to a minimum at period 9. After period 9, the mean discrepancy for all solution concepts experienced a jump and steady increasing trend. While the mean discrepancies for the solution concepts were similar for periods 1 and 2, a marked ranking developed as the number of periods increased, with the egalitarian allocation producing the smallest discrepancy scores, followed by the Shapley value and nucleolus. However, the magnitude of the standard errors for the three solution concepts (as shown in Figure 4.3) indicate that the difference in discrepancy may not be statistically significant.

In order to statistically test the differences in mean percentage deviation scores shown in Tables 4.1, 4.2 and 4.3, Tukey's HSD test was performed following Michener et al. (1980, 1981, 1982, 1983), Tutic, Pfau and Casajus. (2011), and Beimborn (2013). Initially, the data were aggregated across 10 sessions, assuming each round was independent. The egalitarian solution had a significantly lower mean discrepancy score than the Shapley value (d_e = 12.82, d_s = 16.13, $p < 0.05$); and the Shapley value had a significantly lower mean discrepancy score than the nucleolus (d_s = 16.13, d_n = 20.72, $p < 0.05$). Consequently, the egalitarian solution had a significantly lower mean discrepancy score than the nucleolus, $p < 0.05$. Therefore, sufficient evidence was found for aggregated data that egalitarian solution was the most accurate, followed by the Shapley value and nucleolus.

Given the difficulty of reaching a grand coalition demonstrated by the descriptive statistics, it was critical to determine which solution concept carried the highest predictive power when the participants failed to negotiate a grand coalition. In this study, the aggregate data were divided into rounds where the grand coalition was reached, and rounds where partial or singleton coalitions were

reached. On each data subset, the predictive performance of each solution concept was statistically compared.

For the 53 rounds out of 120 where the grand coalition was reached, the egalitarian allocation had a significantly lower mean discrepancy score in comparison with the Shapley value ($d_e = 1.57$, $d_s = 11.28$, $p < 0.05$). Further, the Shapley value had a significantly lower mean discrepancy score compared with the nucleolus solution ($d_s = 11.28$, $d_n = 16.60$, $p < 0.05$). By extension, the egalitarian solution had a significantly lower mean than the nucleolus ($p < 0.05$). The results clearly demonstrate that when the grand coalition was reached, the egalitarian allocation had the highest predictive power by a significant margin. In comparison, both the Shapley value and nucleolus exhibited significantly lower predictive capacity.

For the 67 rounds out of 120 where the participants failed to reach the grand coalition, the egalitarian allocation had a significantly lower mean discrepancy score compared with the nucleolus ($d_e = 21.72$, $d_n = 27.38$, $p < 0.05$). However, interestingly, there was no significant difference in mean discrepancy between the Shapley value and nucleolus allocations. These results suggest that the egalitarian solution was the most accurate solution concept for rounds in which participants failed to reach a grand coalition, but a conclusive rank ordering of the solution concepts could not be determined.

4.5 Round-by-Round Analysis

The inference testing in Section 4.4 assumed that each round was independent with respect to participant behavior. Analysis of round-by-round payoff in Figure 4.1 displayed an increasing trend for player payoff as a function of round number, suggesting that participant learning occurred as each session progressed. To further test the relative performance of the solution concepts, inference testing was performed on a round-by-round basis. The results are shown in Table 4.5.

Table 4.5. Mean differences (MD) and p-values for solution concept comparisons on a round by round basis.

Round	Shapley - Nucleolus		Shapley - Egalitarian		Nucleolus - Egalitarian	
	MD	p-val	MD	p-val	MD	p-val
1	-3.130	0.705	-2.156	0.846	0.947	0.966
2	-4.003	0.598	-0.837	0.977	3.166	0.723
3	-4.511	0.515	3.811	0.620	8.322	0.119
4	-4.283	0.591	2.129	0.876	6.412	0.317
5	-4.183	0.599	3.071	0.756	7.254	0.227
6	-4.613	0.466	3.531	0.636	8.144	0.106
7	-4.745	0.466	3.517	0.654	8.262	0.113
8	-5.159	0.195	5.598	0.149	**10.757**	**0.003**
9	**-5.661**	**0.020**	**6.468**	**0.008**	**12.129**	**0.000**
10	-4.625	0.480	3.681	0.625	8.306	0.108
11	-5.149	0.205	5.604	0.156	**10.753**	**0.003**
12	-5.018	0.348	5.269	0.314	**10.287**	**0.020**

In round 8, the mean discrepancy score of the egalitarian solution was significantly less than the mean score of the nucleolus solution (d_e = 8.31, d_n = 19.06, $p < 0.05$). No significant difference was found between either the egalitarian allocation and the nucleolus, or the Shapley value and the nucleolus. In round 9, the mean discrepancy score of the egalitarian solution was significantly less than the Shapley value, which in turn was significantly less than the nucleolus (d_e = 5.55, d_s = 12.02, d_n = 17.68, $p < 0.05$). As a consequence, the mean discrepancy score of the egalitarian solution was significantly less than the nucleolus, $p < 0.05$.

In round 11, the mean discrepancy score of the egalitarian solution was significantly less than the nucleolus (d_e = 7.81, d_n = 18.56, $p < 0.05$) (d = 18.56 and d = 19.42 respectively). Similarly, in round 12 the egalitarian allocation produced a significantly lower mean discrepancy score than the nucleolus (d_e =

9.13, d_n = 19.42, $p < 0.05$). No significant difference was found in rounds 11 and 12 between the mean discrepancy scores of the Shapley value and nucleolus, or the egalitarian allocation and the Shapley value. In addition, no statistical difference between solution concepts was demonstrated in period 1, in which the Shapley value held a lower mean discrepancy score compared with the egalitarian solution and nucleolus (d_e = 22.27, d_s = 20.11, d_n = 23.24, $p > 0.05$). Additional replications would be required in order to determine if the mean difference in discrepancy score was statistically significant.

4.6 Summary

This chapter has provided a detailed description of the experimental results from the present study. Empirical analysis of participant behavior has highlighted the difficulty of reaching the grand coalition as the number of players increases. Compared with previous 3-player studies, the proportion of laboratory allocations within the core was reduced in the present study. When the data for all sessions was aggregated, inference testing determined a clear rank-ordering of solution concept predictive power, with the egalitarian solution significantly more accurate than the Shapley value, which in turn was more accurate than the nucleolus. This result was mirrored when the experiment rounds were divided into those containing the grand coalition and those where only singleton or partial coalitions were achieved. However, the ordering of solution concept predictive power was not conclusive when the grand coalition could not be reached. In order to correct for learning effects between experiment rounds, inference testing was performed on a round by round basis. The results provided further evidence to support the egalitarian allocation as the most accurate solution concept. Chapter 5 will provide an explanation of the laboratory results and a discussion of the associated policy implications for landscape-scale conservation efforts.

Chapter 5

Discussion and Conclusions

5.1 Outline

Chapter 5 draws on the results to elucidate the key findings of the modeling and laboratory experiments, and their significance with respect to previous studies. The primary objective of the present study was to experimentally test the payoff allocations predicted by the Shapley value, nucleolus and egalitarian solution concepts accurately reflect laboratory allocations, and if so which is the most accurate. The egalitarian allocation was consistently proven to have the highest level of predictive power out of the tested solution concepts. This chapter discusses the significance of these findings by (1) analyzing aggregated data across all sessions; (2) rounds where the grand coalition was reached; (3) rounds where singleton or sub-coalitions were reached; and (4) aggregated data from all sessions on a round-by-round basis. The key findings have been applied to predict the potential behavior of a group of landowners involved in a conservation negotiation on the landscape scale. Based on these findings, policy recommendations for future conservation programs are proposed.

5.2 Core Allocations

The first major concern of the present study was the overall analysis of aggregated data, specifically the formation of allocations within the core. A key finding of the present study is that the 5-player group were only able to reach a core allocation in 44% of rounds. That is, a grand coalition was reached whereby the allocation was un-dominated by any other allocations outside the core. When this result is compared with previous experimental studies, it appears that a core allocation becomes less likely as the number of players increases. For example, Michener, Yuen and Sakurai (1981) reported core allocations in 62.5% of rounds with a 3-player cooperative game (pp. 84). Similarly, Michener and Yuen (1982) found core allocations in 60.94% to 79.17% of rounds for a 3-player game,

depending on treatment (pp. 64). When the number of players was increased to 4, the proportion of allocations within the core was found to be between 57.5% and 62.5% depending on treatment (Michener, Ginsberg and Yuen, 1979, pp. 274). Taken collectively in the light of the present study's findings, these results imply that the difficulty of reaching a core allocation rises as the number of players in the game increases. The implications for real-world conservation negotiations, which may involve upwards of 20 landowners, is that reaching an allocation within the core is unlikely in the absence of external regulation or additional incentives for coalition formation.

The only previous studies that fail to conform to this pattern are those conducted by Michener, Yuen and Geishenker (1980), who found that only 11.46% of rounds resulted in a core allocation for a 4-player game (pp. 512); and Marchetti and Serra (2003), who demonstrated that a core allocation was generated in only 37% of rounds in a 3-player game (pp. 6). In the case of the study by Michener, Yuen and Geishenker (1980) it is probable that a minimized core size was responsible for the very small percentage of core allocation, the aim being to test core magnitude as an independent variable in solution concept performance. This is in contrast to the present study, where the aim was to use a suitable core size for the purposes of laboratory experiments. Secondly, according to Marchetti and Serra (2003), a lower than expected proportion of observed allocations in the core was the result of external factors such as altruism or reciprocity. This finding suggests that the social preferences of individual players can influence the proportion of core allocations formed, regardless of core size or number of players.

5.3 Solution Concept Accuracy

When all data for the present study was aggregated, the key finding was that the egalitarian allocation dominated the Shapley value and nucleolus allocations with respect to predictive accuracy. Indeed, the relative predictive power of the three solution concepts can be completely ordered, with the egalitarian allocation providing greater accuracy than the Shapley value, which in turn

provided greater accuracy than the nucleolus. The superior predictive performance of the egalitarian allocation is in contrast with the findings of Michener, Ginsberg and Yuen (1979), who demonstrated that the Shapley value offered the highest accuracy. In Michener, Ginsberg and Yuen's (1979) study, the presence of unconstrained communication could have contributed to the lower predictive power of the egalitarian allocation. That is, unconstrained communication has the potential to facilitate players revealing individual private valuations, reducing the likelihood of forming an equal distribution of payoff. In the present study, communication between participants was carefully controlled in the light of findings by Bolton, Chatterjee and McGinn (2003), who determined that public communication reduced the incidence of core allocations.

However, the relative predictive power of the Shapley value and nucleolus in the present study is consistent with the results of previous research by Michener, Ginsberg and Yuen (1979), Michener and Yuen (1982) and Marchetti and Serra (2003), who all demonstrated that the Shapley value offered a higher degree of accuracy in comparison to the nucleolus. Taken collectively, the nucleolus appears to be consistently less accurate than the Shapley value and the egalitarian solution.

Given the difficulty participants experienced reaching an allocation within the core, the present study separately analyzed those rounds where the grand coalition was reached versus those rounds where participants failed to reach the grand coalition. This novel approach has not been previously described in the literature. In rounds where the core was reached, the egalitarian allocation offered significantly higher predictive power than the Shapley value, which in turn was more accurate than the nucleolus. This was consistent with the results for aggregated data. Indeed, in the majority of rounds where the core was reached, participants closely adhered to an equal allocation of payoff. In rounds where only singleton or partial coalitions were reached, the egalitarian allocation was the most accurate solution concept, but no significant difference in predictive power could be found between the Shapley value and nucleolus. Thus, the egalitarian allocation offered significantly greater predictive power

than either the Shapley value or nucleolus, regardless of whether participants were able to reach the grand coalition. However, the inconsistent lexicographical ordering of the Shapley value and nucleolus solutions where participants failed to reach the grand coalition necessitated a round by round analysis.

5.4 Predictive Performance on a Round-by-Round Basis

When the data for all sessions were considered on a round by round basis, a key finding was that only the later rounds resulted in a significant difference in accuracy between the egalitarian allocation, Shapley value and nucleolus. Specifically, no differences in predictive power could be demonstrated up to round 8 of the experiment. In these later rounds, the egalitarian allocation offered significantly greater accuracy than either the Shapley value or the nucleolus. Further, if allocations for period 1 are considered as representative of a single-shot interaction between participants, the present study demonstrates that there is no difference in predictive power between the solution concepts. However, when participants are exposed to repeated interactions, this study implies that the egalitarian allocation offers the highest level of predictive power.

5.5 Summary and Conclusions

The critical findings of the present study are as follows. Firstly, as the number of players increases, the likelihood of reaching a grand coalition decreases. Therefore, for a real-world landscape conservation program, the probability of forming a grand coalition consisting of all landowners in a region is small. When the data was aggregated, the egalitarian allocation provided the best approximation of laboratory allocations. For rounds where the grand coalition was reached, the egalitarian solution provided an even more accurate prediction of participant behavior. By comparison, both the Shapley value and nucleolus solution concepts were strictly dominated by the egalitarian allocation, although evidence suggests that the Shapley value offers significantly higher predictive capability than the nucleolus. Finally, a significant difference in accuracy only

becomes apparent in later experiment rounds, suggesting that participant learning influences the predictive power of the egalitarian allocation.

Based on these findings the following policy recommendations can be suggested. As reaching a grand coalition is unlikely for conservation programs that consist of large groups of landowners, regulators should provide additional incentives for coalition formation. Secondly, participants are likely to allocate equally in repeated interactions regardless of the value of operating individually. If this is not a desirable outcome for a conservation program, additional controls beyond unrestricted negotiation between landowners may be required. Thirdly, in conservation programs consisting of a single-shot interaction, the allocation preferences of groups of landowners are not clear and warrant further investigation. In addition, directions for further research could include investigating the effect of participant social preferences on the proportion of core allocations and solution concept predictive power. Finally, the effect of communication availability on the predictive accuracy of the egalitarian allocation is another potential area for further examination.

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